WBSU WORKSHOP ON GEOA (SEM 2 CORE 04T)

(Study Materials)

- **Logarithms**

A logarithm of a number is the power to which a given base must be raised to obtain that number. The power is sometimes called the exponent. In other words, if $b^y = x$ then y is the logarithm of x to base b. For example, if $2^4 = 16$, then 4 is the logarithm of 16 with the base as 2. We can write it as $4 = \log_2 = 16$.

Learn more about Exponent here in detail.

$$b^y = x$$

$$\text{or} \quad y \log b = \log x$$

$$\text{or} \quad y = \log_b x$$

Here, $y > 0$, $b > 0$, and $b \neq 1$.

- **Logarithmic Laws and Properties**

Theorem 1

The logarithm of the product of two numbers say x, and y is equal to the sum of the logarithm of the two numbers. The base should be the same for both the numbers.

$\log_b (x\,y) = \log_b x + \log_b y$

Proof: Let $\log_b x = p$ such that $b^p = x$ … (i), and

$\log_b y = q$ such that $b^q = y$ … (ii)

Multiplying (i), and (ii), we have

$b^p \times b^q = x \times y = b^{(p+q)}$ [from the law of indices]

Taking log on both sides, we have,

$\log_b x\,y = p + q = \log_b x + \log_b y$.

Theorem 2

The division of the two numbers is the antilog of the difference of logarithm of the two numbers. The base should be the same for both the numbers.

$\log x/y = \log x - \log y$

1. **Proof:** Let, $\log_b x = p$ such that $b^p = x$ … (i), and

$\log_b y = q$ such that $b^q = y$ … (ii)

Dividing (i) by (ii), we have

$x/y = b^p/b^q = b^{(p-q)}$ [from the law of indices]

Taking log on both sides, we have,

$\log x/y = p - q = \log x - \log y$

Theorem 3

The logarithm of a number to any other base can be determined by the logarithm of the same number to any given base. Mathematically, the relation is

$\log_a x = \log_b x \times \log_a b$

$\Rightarrow \log_b x = \log_a x / \log_a b$

Proof: Let, $\log_a x = p$, $\log_b x = q$, and $\log_a b = r$. From the definition of logarithms, we have

$a^p = x = b^q$, and $a^r = b$.

$b^q = x$ can be written as $(a^r)^q = a^{rq} = x$.

Since, $a^p = b^q = a^{rq} = x$. Comparing the powers, we have

$p = rq$

or, $\log_a x = \log_a b \times \log_b x$

or, $\log_b x = \log_a x / \log_a b$.

Theorem 4

The logarithm of a number raised to a power is equal to the index of the power multiplied by the logarithm of the number. The base is the same in both the conditions.

$\log_b x^n = n \log_b x$.

Proof: Let $\log_b x = p$ so that $b^p = x$. Raising both sides to power n, we have

$(b^p)^n = x^n \Rightarrow b^{pn} = x^n$

Taking log on both the sides, we have $\log_b x^n = pn$

or, $\log_b x^n = n \log_b x$.

- **Logarithmic Table**

It is not always necessary to find the logarithm of a number by mere calculation. We can also use logarithm table to find the logarithm of a number. The logarithm of a number comprises of two parts. The whole part is the characteristics and the decimal part is the mantissa.

Characteristic

The whole part or the integral part of a number is the characteristic. The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of the digits to the left of the decimal point in the given number. If the number is less than one, then the characteristic is negative and is one more than the number of zeros to the right of the decimal point.

For Example

Number	Characteristic
4	0 [one less than the number of digits to the left of the decimal point].
21	1
111	2
0.1	– 1 [one more than the number of zeros on the right immediately after the decimal point].
0.025	– 2
0.000010	– 5

The logarithm of a number having 'n' zeros immediately after the decimal is $-(n+1)$ + the decimal.

Mantissa

The decimal part of the number logarithm of a number is the mantissa. A mantissa is always a positive quantity. The negative mantissa should always be converted into a positive one. For example, $-5.2592 = -6 + (1 - 0.2592) = \bar{6} + 0.7428$

- **Anti-Logarithms (Antilog)**

The anti-logarithm of a number is the inverse process of finding the logarithms of the same number. If x is the logarithm of a number y with a given base b, then y is the anti-logarithm of (antilog) of x to the base b.

$$\text{If } \log_b y = x \quad \text{then, } y = \text{antilog}_b\, x$$

Natural Logarithms and Anti-Logarithms have their base as 2.7183. The Logarithms and Anti-Logarithms with base 10 can be converted into natural Logarithms and Anti-Logarithms by multiplying it by 2.303.

Anti-Logarithmic Table

To find the anti-logarithm of a number we use an anti-logarithmic table. Below are the steps to find the antilog.

- The first step is to separate the characteristic and the mantissa part of the number.
- Use the antilog table to find a corresponding value for the mantissa. The first two digits of the mantissa work as the row number and the third digit is equal to the column number. Note this value.
- The antilog table also includes columns which provide the mean difference. For the same row of the mantissa, the column number in the mean difference is equal to the fourth digit. Note this value.
- Add the values so obtained.
- In the characteristic add one. This value shows the place to put the decimal point. The decimal point is inserted after that many digits from the left.

SCIENTIFIC NOTATION

Scientific notation is a standard way of writing very large and very small numbers so that they're easier to both compare and use in computations. To write in scientific notation, follow the form

$M \times 10^a$

where M is a number between 1 and 10, but not 10 itself, and a is an integer (positive or negative number).

You move the decimal point of a number until the new form is a number from 1 up to 10 (M), and then record the exponent (a) as the number of places the decimal point was moved. Whether the power of 10 is positive or negative depends on whether you move the decimal to the right or to the left. Moving the decimal to the right makes the exponent negative; moving it to the left gives you a positive exponent.

To see an exponent that's positive, write 312,000,000,000 in scientific notation:

Move the decimal place to the left to create a new number from 1 up to 10.

Where's the decimal point in 312,000,000,000? Because it's a whole number, the decimal point is understood to be at the end of the number: 312,000,000,000.

So, N = 3.12.

Determine the exponent, which is the number of times you moved the decimal.

In this example, you moved the decimal 11 times; also, because you moved the decimal to the left, the exponent is positive. Therefore, a = 11, and so you get

Put the number in the correct form for scientific notation

To see an exponent that's negative, write .00000031 in scientific notation.

Move the decimal place to the right to create a new number from 1 up to 10.

So, N = 3.1.

Determine the exponent, which is the number of times you moved the decimal.

In this example, you moved the decimal 7 times; also, because you moved the decimal to the right, the exponent is negative. Therefore, a = –7, and so you g

Convert 4.2×10^{-7} to decimal notation.

Since the exponent on 10 is negative, I am looking for a small number. Since the exponent is a seven, I will be moving the decimal point seven places. Since I need to move the point to get a small number, I'll be moving it to the left.

The answer is 0.000 000 42

Convert 0.000 000 005 78 to scientific notation.

This is a small number, so the exponent on 10 will be negative. The first "interesting" digit in this number is the 5, so that's where the decimal point will need to go. To get from where it is to right after the 5, the decimal point will need to move nine places to the right. (Count 'em out, if you're not sure!)

Then the power on 10 will be a negative 9, and the answer is 5.78×10^{-9}

Convert 93,000,000 to scientific notation.

This is a large number, so the exponent on 10 will be positive. The first "interesting" digit in this number is the leading 9, so that's where the decimal point will need to go. To get from where it is to right after the 9, the decimal point will need to move seven places to the left.

Then the power on 10 will be a positive 7, and the answer is 9.3×10^{7}

Log x = Characteristic of x + Mantissa of x

1.How to determine the characteristic of log x :

If x>1, then count the digits on the left of the decimal point; if the number of digits is y, then the characteristic is (y-1).

If 0<x<1, then count the number of zeroes appearing in the right side of the decimal point; if the number of zeros is z, then the characteristic is (z+1).

2. How to determine the mantissa of log x :

As mentioned earlier, the mantissa has to be read from a standard log table. Log tables consist of rows that go from 10, 11, up to 99. The columns have values 0, 1, 2, up to 9. Beyond the 10 columns, there is another column which is known as the mean difference. For determining the mantissa, a particular row has to be read off and the mean difference has to be added from the table.

The following has to be remembered:

- Mantissa usually consists of a four digit number, and it comes after the decimal point.
- Mantissa is a non–negative real number, which is less then 1.
- While determining the mantissa, the decimal point of the number has to be ignored.
- Most of the log tables give values of mantissa up to four digits only. For more than a four digit mantissa, we have to round off the last digit.
- Number with same sequence of digits has same mantissa.

Example 1: Find the log of 500.2

Characteristic = 2

For mantissa, read from the table a number 5002. From the rows, choose 50, and read off from the number under the column 0. The number given in the log tables is 6990. Now read, in the same row, the mean difference under 2. This number is given as 2.

Mantissa = 6990 + 2 = 6992

Thus log 500.2 = Characteristic of 500.2 + Mantissa of 500.2

= 2 + 0.6992

= 2.6992

Example 2: Find the log of 72.98

Characteristic = 1

For mantissa, read from the table a number 7298. From the rows, choose 72, and read off from the number under the column 9. The number given in the log tables is 8627. Now read, in the same row, the mean difference under 8. This number is given as 5.

Mantissa = 8627 + 5 = 8632

Thus log 72.98 = Characteristic of 72.98 + Mantissa of 72.98

= 1 + 0.8632

= 1.8632

Example 3: Find the log of 0.0009887

Characteristic = –4.

For mantissa, read from the table a number 9887. From the rows, choose 98, and read off from the number under the column 8. The number given in the log tables is 9948. Now read, in the same row, the mean difference under 7. This number is given as 3.

Mantissa = 9948 + 3 = 9951

Thus log 0.0009887 = Characteristic of 0.0009887 + Mantissa of 0.0009887

= – 4 + 0.9951

= –3.0049 (The log of 0.009887 is also written as .9951, although its value is – 3.0049)

Example 4: Find the log of 0.1234

Characteristic = – 1

For mantissa, read from the table a number 1234. From the rows, choose 12, and read off from the number under the column 3. The number given in the log tables is 0899. Now read, in the same row, the mean difference under 4. This number is given as 14.

Mantissa = 0899 + 14 = 0913

Thus log 0.1234 = Characteristic of 0.1234 + Mantissa of 0.1234

= – 1 + 0. 0913

= – 0.9087 or .0913

The log of 1.234 will be 0.0913

The log of 12.34 will be 1.0913

The log of 123.4 will be 2.0913

The log of 1234 will be 3.0913

To keep consistency, the log of 0.1234 is written as .0913, (although its value is – 0.9087).

The log of 12344 will be as follows: this is a five digit number, so the last that is the fifth digit will have to be rounded off. The fifth digit is 4, which is less than 5. So take the last digit is 0. Thus the mantissa of 12344 will be same as the mantissa for 1234.

log 12344 = 4.0913.

The log of 12346 will be as follows: the last digit 6 is rounded off as 1 and is added to the second last number 4. Thus the last digit becomes 4 + 1 = 5. So we have to find the mantissa for 1235, which is 0899 + 15 = 08914.

log 12346 = 4.08914.

Antilogarithm

Antilogarithm is the exact opposite of logarithm of a number.

If x = log b, then antilog (x) = b.
Remember that $antilog_a (x) = b$.

Antilog table for base 10 is readily available. Antilog tables are used for determining the inverse value of the mantissa.

From the characteristic, the position of the decimal point can be determined.

Antilog tables consist of rows that go from .00, .01, up to .99. The columns have values 0, 1, 2, up to 9. Beyond the 10 columns, there is another column which is known as the mean difference. For determining the antilog of the numbers after the decimal point, a particular row has to be read off and the mean difference has to be added from the table.

Example 1 : Find the antilog of 2.6992

The number before the decimal point is 2, so the decimal point will be after the first 3 digits.

From the antilog table, read off the row for .69 and column of 9; the number given in the table is 5000. The mean difference in the same row and under the column 2 is 2. To get the inverse of mantissa add 5000 + 2 = 5002

Now place a decimal point after the first 3 digits and you get the number 500.2

Thus antilog 2.6992 = 500.2

Example 2 : Find the antilog of – 1.9087

Convert – 1.9087 in bar notation as follows:
characteristic of – 1.9087 = = –2
mantissa of –1.9087 = – 1.9087– (–2) = 0.0913 +1 −1
so –0.9087 = .0913

The number before the decimal point is 0, the number of zeroes after the decimal point is one.

From the antilog table, read off the row for .09 and column of 1; the number given in the table is 1233. The mean difference in the same row and under the column 3 is 1. To get the inverse of mantissa add 1233 + 1 = 1234

Now place a decimal point before the 1234 followed by a zero and you get the number 0.01234. $\frac{1234}{10^{5}}$ = 0.01234

Applications
We will now see how logarithms and antilogarithms of numbers are useful for calculations which are complicated or have very large/small numbers.

Example 1 : Find 80.92 * 19.45

Let x = 80.92 * 19.45

Use the log function on both the sides.

log x = log (80.92 * 19.45)