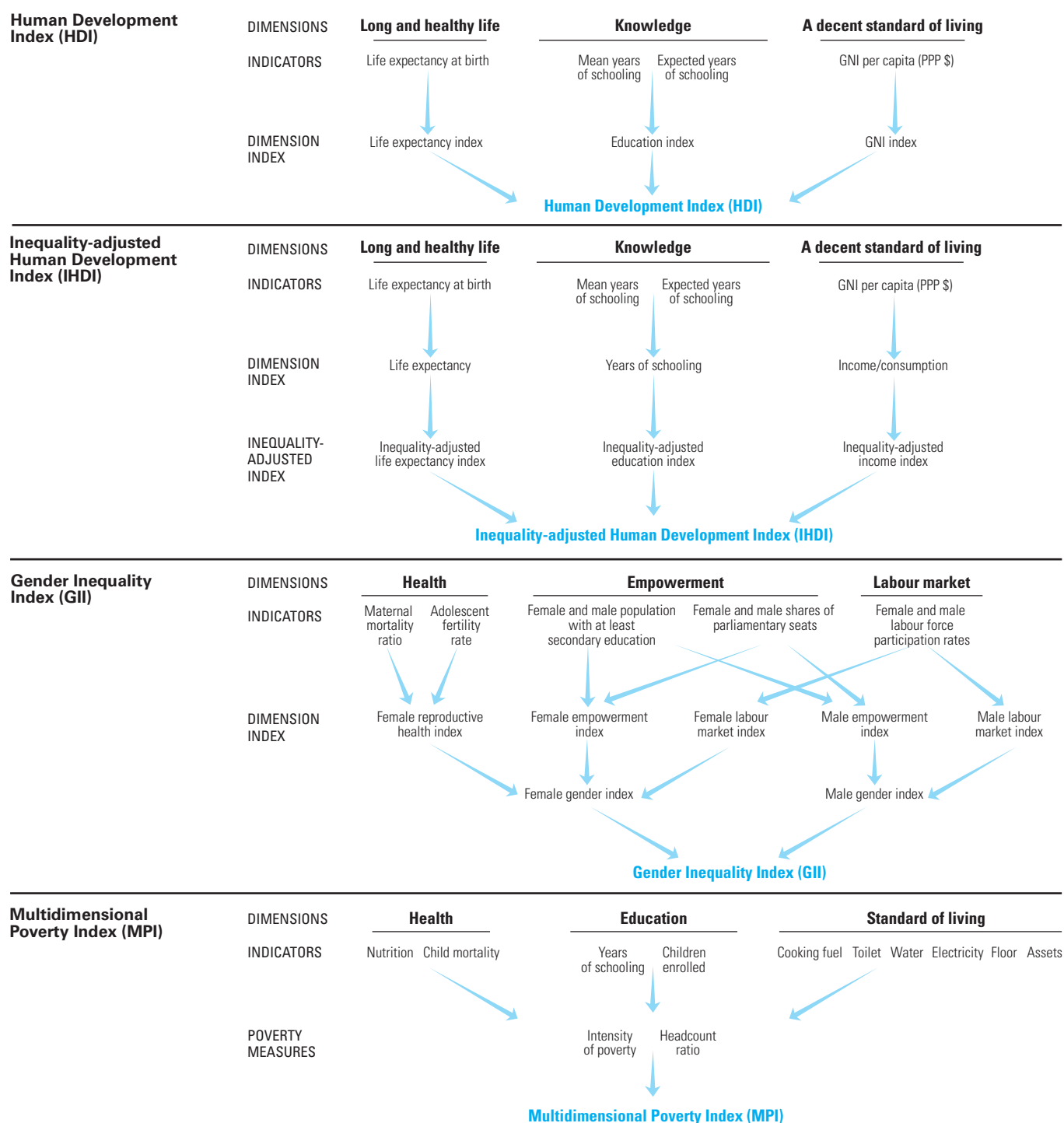


Technical notes

Calculating the human development indices—graphical presentation



Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of key dimensions of human development. It measures the average achievements in a country in three basic dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices from each of these three dimensions. For a full elaboration of the method and its rationale, see Klugman, Rodriguez and Choi (2011). This technical note describes the steps to calculate the HDI, data sources and the methodology used to express income.

Steps to calculate the Human Development Index

There are two steps to calculating the HDI.

Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators into indices between 0 and 1. The maximums are the highest observed values in the time series (1980–2012). The minimum values can be appropriately conceived of as subsistence values. The minimum values are set at 20 years for life expectancy, at 0 years for both education variables and at \$100 for per capita gross national income (GNI). The low value for income can be justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, not captured in the official data.

Goalposts for the Human Development Index in this Report

Indicator	Observed maximum	Minimum
Life expectancy (years)	83.6 (Japan, 2012)	20.0
Mean years of schooling	13.3 (United States, 2010)	0
Expected years of schooling	18.0 (capped at)	0
Combined education index	0.971 (New Zealand, 2010)	0
GNI per capita (PPP \$)	87,478 (Qatar, 2012)	100

Having defined the minimum and maximum values, the subindices are calculated as follows:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \quad (1)$$

For education, equation 1 is applied to each of the two sub-components, then a geometric mean of the resulting indices is created and finally, equation 1 is reapplied to the geometric mean of the indices using 0 as the minimum and the highest geometric

mean of the resulting indices for the time period under consideration as the maximum. This is equivalent to applying equation 1 directly to the geometric mean of the two subcomponents.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000). Thus, for income the natural logarithm of the actual, minimum and maximum values is used.

Step 2. Aggregating the subindices to produce the Human Development Index

The HDI is the geometric mean of the three dimension indices:

$$(I_{\text{Life}}^{\frac{1}{3}} \cdot I_{\text{Education}}^{\frac{1}{3}} \cdot I_{\text{Income}}^{\frac{1}{3}}). \quad (2)$$

Example: Ghana

Indicator	Value
Life expectancy at birth (years)	64.6
Mean years of schooling	7.0
Expected years of schooling	11.4
GNI per capita (PPP \$)	1,684

Note: Values are rounded.

$$\text{Life expectancy index} = \frac{64.6 - 20}{83.6 - 20} = 0.701$$

$$\text{Mean years of schooling index} = \frac{7.0 - 0}{13.3 - 0} = 0.527$$

$$\text{Expected years of schooling index} = \frac{11.4 - 0}{18.0 - 0} = 0.634$$

$$\text{Education index} = \frac{\sqrt{0.527 \cdot 0.634} - 0}{0.971 - 0} = 0.596$$

$$\text{Income index} = \frac{\ln(1,684) - \ln(100)}{\ln(87,478) - \ln(100)} = 0.417$$

$$\text{Human Development Index} = \sqrt[3]{0.701 \cdot 0.596 \cdot 0.417} = 0.558$$

Data sources

- Life expectancy at birth: UNDESA (2011)
- Mean years of schooling: Barro and Lee (2011) and HDRO updates based on UNESCO Institute for Statistics (2012) data on education attainment using the methodology outlined in Barro and Lee (2010)

- Expected years of schooling: UNESCO Institute for Statistics (2012)
- GNI per capita: World Bank (2012a), IMF (2012), UNSD (2012a) and UNDESA (2011)

Methodology used to express income

GNI is traditionally expressed in current monetary terms. To make GNI comparable across time, GNI is converted from current to constant terms by taking the value of nominal GNI per capita in purchasing power parity (PPP) terms for the base year (2005) and building a time series using the growth rate of real GNI per capita, as implied by the ratio of current GNI per capita in local currency terms to the GDP deflator.

Official PPPs are produced by the International Comparison Program (ICP), which periodically collects thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2005 and covers 146 countries. The 2011 round will produce new estimates by the end of 2013. The World Bank produces estimates for years other than the ICP benchmark based on inflation relative to the United States. Because other international organizations—such as the World Bank and the International Monetary Fund (IMF)—quote the base year in terms of the ICP benchmark, the HDRO does the same.

To obtain the income value for 2012, IMF-projected GDP growth rates (based on growth in constant terms) are applied to the most recent GNI values. The IMF-projected growth rates are calculated in local currency terms and constant prices rather than in PPP terms. This avoids mixing the effects of the PPP conversion with those of real growth of the economy.

Estimating missing values

For a small number of countries that were missing one out of four indicators, the HDRO estimated the missing value using cross-country regression models. The details of the models used are available at <http://hdr.undp.org/en/statistics/understanding/issues/>.

In this Report, the PPP conversion rates were estimated for Cuba and Occupied Palestinian Territory; expected years of schooling were estimated for Haiti, Liberia, Federated States of Micronesia, Palau, Papua New Guinea, Sierra Leone, South Africa, Tanzania, Turkmenistan, Zambia and Zimbabwe; and mean years of schooling were estimated for Antigua and Barbuda, Bahamas, Cape Verde, Eritrea, Grenada, Kiribati, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Solomon Islands and Vanuatu. The total number of countries with an HDI value calculated for 2012 remains 187.

Technical note 2. Inequality-adjusted Human Development Index

The Inequality-adjusted Human Development Index (IHDI) adjusts the Human Development Index (HDI) for inequality in the distribution of each dimension across the population. It is based on a distribution-sensitive class of composite indices proposed by Foster, Lopez-Calva and Szekely (2005), which draws on the Atkinson (1970) family of inequality measures. It is computed as a geometric mean of geometric means, calculated across the population for each dimension separately (for details, see Alkire and Foster 2010).

The IHDI accounts for inequalities in HDI dimensions by “discounting” each dimension’s average value according to its level of inequality. The IHDI equals the HDI when there is no inequality across people but falls further below the HDI as inequality rises. In this sense, the IHDI is the actual level of human development (taking into account inequality), while the HDI can be viewed as an index of the “potential” human development that could be achieved if there was no inequality. The “loss” in potential human development due to inequality is the difference between the HDI and the IHDI and is expressed as a percentage.

Data sources

Since the HDI relies on country-level aggregates such as national accounts for income, the IHDI must draw on alternative sources of data to obtain insights into the distribution. The distributions have different units—life expectancy is distributed across a hypothetical cohort, while years of schooling and income are distributed across individuals.

Inequality in the distribution of HDI dimensions is estimated for:

- Life expectancy, using data from abridged life tables provided by UNDESA (2011). This distribution is grouped in age intervals (0–1, 1–5, 5–10, ... , 85+), with the mortality rates and average age at death specified for each interval.
- Mean years of schooling, using household survey data harmonized in international databases, including the Luxembourg Income Study, Eurostat’s European Union Survey of Income and Living Conditions, the World Bank’s International Income Distribution Database, the United Nations Children’s Fund’s Multiple Indicators Cluster Survey, ICF

Macro's Demographic and Health Survey and the United Nations University's World Income Inequality Database.

- Disposable household income or consumption per capita using the above listed databases and household surveys—or for a few countries, income imputed based on an asset index matching methodology using household survey asset indices (Harttgen and Vollmer 2011).

A full account of data sources used for estimating inequality in 2012 is available at <http://hdr.undp.org/en/statistics/ihdi/>.

Steps to calculate the Inequality-adjusted Human Development Index

There are three steps to calculating the IHDI.

Step 1. Measuring inequality in the dimensions of the Human Development Index

The IHDI draws on the Atkinson (1970) family of inequality measures and sets the aversion parameter ϵ equal to 1.¹ In this case the inequality measure is $A = 1 - g/\mu$, where g is the geometric mean and μ is the arithmetic mean of the distribution. This can be written as:

$$A_x = 1 - \frac{\sqrt[n]{X_1 \dots X_n}}{\bar{X}} \quad (1)$$

where $\{X_1, \dots, X_n\}$ denotes the underlying distribution in the dimensions of interest. A_x is obtained for each variable (life expectancy, mean years of schooling and disposable income or consumption per capita).²

The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes. Sensitivity analysis of the IHDI is given in Kovacevic (2010).

Step 2. Adjusting the dimension indices for inequality

The mean achievement in an HDI dimension, $\bar{X}$, is adjusted for inequality as follows:

$$\bar{X} \cdot (1 - A_x) = \sqrt[n]{X_1 \dots X_n}.$$

Thus the geometric mean represents the arithmetic mean reduced by the inequality in distribution.

The inequality-adjusted dimension indices are obtained from the HDI dimension indices, I_x , by multiplying them by $(1 - A_x)$, where A_x , defined by equation 1, is the corresponding Atkinson measure:

$$I_x^* = (1 - A_x) \cdot I_x.$$

The inequality-adjusted income index, I_{Income}^* , is actually an adjusted index of the unlogged income values, I_{Income} . This enables the IHDI to account for the full effect of income inequality.

Step 3. Combining the dimension indices to calculate the Inequality-adjusted Human Development Index

The IHDI is the geometric mean of the three dimension indices adjusted for inequality. First, the IHDI that includes the unlogged income index ($IHDI^*$) is calculated:

$$IHDI^* = \sqrt[3]{I_{Life}^* \cdot I_{Education}^* \cdot I_{Income}^*} = \sqrt[3]{(1 - A_{Life}) \cdot I_{Life} \cdot (1 - A_{Education}) \cdot I_{Education} \cdot (1 - A_{Income}) \cdot I_{Income}}.$$

The HDI based on unlogged income index (HDI^*) is then calculated:

$$HDI^* = \sqrt[3]{I_{Life} \cdot I_{Education} \cdot I_{Income}^*}.$$

The percentage loss in the HDI^* due to inequalities in each dimension is calculated as:

$$Loss = 1 - \frac{IHDI^*}{HDI^*} = 1 - \sqrt[3]{(1 - A_{Life}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})}.$$

Assuming that the percentage loss due to inequality in income distribution is the same for both average income and its logarithm, the IHDI is then calculated as:

$$IHDI = \left(\frac{IHDI^*}{HDI^*} \right) \cdot HDI = \sqrt[3]{(1 - A_{Life}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})} \cdot HDI.$$

Notes on methodology and caveats

The IHDI is based on the Atkinson index, which satisfies subgroup consistency. This ensures that improvements or deteriorations in the distribution of human development within a

certain group of society (while human development remains constant in the other groups) will be reflected in changes in the overall measure of human development. This index is also path independent, which means that the order in which data are aggregated across individuals, or groups of individuals, and across dimensions yields the same result—so there is no need to rely on a particular sequence or a single data source. This allows estimation for a large number of countries.

The main disadvantage is that the IHDI is not association sensitive, so it does not capture overlapping inequalities. To make the measure association sensitive, all the data for each individual must be available from a single survey source, which is not currently possible for a large number of countries.

Example: Indonesia

Indicator	Indicator	Dimension index	Inequality measure ^a (A1)	Inequality-adjusted index
Life expectancy (years)	69.8	0.783	0.168	$(1 - 0.168) \cdot 0.783 = 0.652$
Mean years of schooling	5.8	0.439		
Expected years of schooling	12.9	0.714		
Education index		0.577	0.204	$(1 - 0.204) \cdot 0.577 = 0.459$
Logarithm of gross national income	8.33	0.550		
Gross national income (PPP \$)	4,154	0.046	0.177	$(1 - 0.177) \cdot 0.046 = 0.038$
		Human Development Index	Inequality-adjusted Human Development Index	Loss (%)
HDI with unlogged income		$\sqrt[3]{0.783 \cdot 0.577 \cdot 0.046} = 0.275$	$\sqrt[3]{0.652 \cdot 0.459 \cdot 0.038} = 0.225$	$100 \cdot \frac{1 - 0.225}{0.275} = 18.3$
HDI		$\sqrt[3]{0.783 \cdot 0.577 \cdot 0.550} = 0.629$	$(0.225 / 0.275) \cdot 0.629 = 0.514$	

Note: Values are rounded.

a. Obtained from micro data: from life tables (UNDESA 2011) for life expectancy and from the World Bank's International Income Distribution Database for education and income distributions (the 2009 Survei Sosial Ekonomi Nasional was used for Indonesia).

Technical note 3. Gender Inequality Index

The Gender Inequality Index (GII) reflects gender-based disadvantages in three dimensions—reproductive health, empowerment and the labour market—for as many countries as data of reasonable quality allow. The index shows the loss in potential human development due to inequality between female and male achievements in these dimensions. It varies between 0, where women and men fare equally, and 1, where either gender fares as poorly as possible in all measured dimensions.

It is computed using the association-sensitive inequality measure suggested by Seth (2009). The index is based on the general mean of general means of different orders—the first aggregation is by the geometric mean across dimensions; these means, calculated separately for women and men, are then aggregated using a harmonic mean across genders.

Data sources

- Maternal mortality ratio (*MMR*): WHO and others (2012)
- Adolescent fertility rate (*AFR*): UNDESA (2011)
- Share of parliamentary seats held by each sex (*PR*): IPU (2012)
- Attainment at secondary and higher education (*SE*) levels: Barro and Lee (2011) and UNESCO Institute for Statistics (2012)
- Labour market participation rate (*LFPR*): ILO (2012)

Steps to calculate the Gender Inequality Index

There are five steps to calculating the GII.

Step 1. Treating zeros and extreme values

Because a geometric mean cannot be computed from a zero value, a minimum value of 0.1% is set for all component indicators. This implies that the maximum value for the maternal mortality ratio is truncated at 1,000 deaths per 100,000 births and that the female parliamentary representation of countries reporting zero is coded as 0.1%. Truncating the maternal mortality ratio can be justified by the normative assumption that countries with a maternal mortality ratio exceeding 1,000 do not differ in their inability to create conditions and support for maternal health. And even in countries without female members of the national parliament, women have some political influence.

Similarly, it is assumed that countries with 1–10 deaths per 100,000 live births are performing at essentially the same level and that differences are random; thus, they are all assigned a value of 10. Sensitivity analysis of the GII is given in Gaye and others (2010).

Step 2. Aggregating across dimensions within each gender group, using geometric means

Aggregating across dimensions for each gender group by the geometric mean makes the GII association sensitive (see Seth 2009).

For women and girls, the aggregation formula is

$$G_F = \sqrt[3]{\left(\frac{10}{MMR} \cdot \frac{1}{AFR}\right)^{\frac{1}{2}} \cdot (PR_F \cdot SE_F)^{\frac{1}{2}} \cdot LFPR_F}, \quad (1)$$

and for men and boys the formula is

$$G_M = \sqrt[3]{1 \cdot (PR_M \cdot SE_M)^{\frac{1}{2}} \cdot LFPR_M}.$$

The rescaling by 0.1 of the maternal mortality ratio in equation 1 is needed to account for the truncation of the maternal mortality ratio minimum at 10.

Step 3. Aggregating across gender groups, using a harmonic mean

The female and male indices are aggregated by the harmonic mean to create the equally distributed gender index

$$HARM(G_F, G_M) = \left[\frac{(G_F)^{-1} + (G_M)^{-1}}{2} \right]^{-1}.$$

Using the harmonic mean of geometric means within groups captures the inequality between women and men and adjusts for association between dimensions.

Step 4. Calculating the geometric mean of the arithmetic means for each indicator

The reference standard for computing inequality is obtained by aggregating female and male indices using equal weights (thus treating the genders equally) and then aggregating the indices across dimensions:

$$G_{\bar{F}, \bar{M}} = \sqrt[3]{\overline{Health} \cdot \overline{Empowerment} \cdot \overline{LFPR}}$$

$$\text{where } \overline{Health} = \left(\sqrt{\frac{10}{MMR} \cdot \frac{1}{AFR}} + 1 \right) / 2,$$

$$\overline{Empowerment} = \left(\sqrt{PR_F \cdot SE_F} + \sqrt{PR_M \cdot SE_M} \right) / 2, \text{ and}$$

$$\overline{LFPR} = \frac{LFPR_F + LFPR_M}{2}.$$

$\overline{Health}$ should not be interpreted as an average of corresponding female and male indices but as half the distance from the norms established for the reproductive health indicators—fewer maternal deaths and fewer adolescent pregnancies.

Step 5. Calculating the Gender Inequality Index

Comparing the equally distributed gender index to the reference standard yields the GII,

$$1 - \frac{HARM(G_F, G_M)}{G_{\bar{F}, \bar{M}}}.$$

Example: Brazil

	Health		Empowerment		Labour market
	Maternal mortality ratio	Adolescent fertility rate	Parliamentary representation	Attainment at secondary and higher education	Labour market participation rate
Female	56.0	76.0	0.096	0.488	0.596
Male	na	na	0.904	0.463	0.809
$\frac{F+M}{2}$	$\sqrt{\left(\frac{10}{56}\right) \cdot \left(\frac{1}{76}\right)} + 1$		$\frac{\sqrt{0.096 \cdot 0.488} + \sqrt{0.904 \cdot 0.463}}{2}$		$\frac{0.596 + 0.809}{2}$
	$= 0.524$		$= 0.432$		$= 0.703$

na is not applicable.

Using the above formulas, it is straightforward to obtain:

$$G_F = 0.185 = \sqrt[3]{\sqrt{\frac{10}{56} \cdot \frac{1}{76}} \cdot \sqrt{0.096 \cdot 0.488} \cdot 0.596}$$

$$G_M = 0.812 = \sqrt[3]{1 \cdot \sqrt{0.904 \cdot 0.463} \cdot 0.809}$$

$$HARM(G_F, G_M) = 0.302 = \left[\frac{1}{2} \left(\frac{1}{0.185} + \frac{1}{0.812} \right) \right]^{-1}$$

$$G_{\bar{F}, \bar{M}} = 0.546 = \sqrt[3]{0.524 \cdot 0.432 \cdot 0.703}$$

$$GII = 1 - (0.302/0.546) = 0.447.$$

Technical note 4. Multidimensional Poverty Index

The Multidimensional Poverty Index (MPI) identifies multiple deprivations at the individual level in education, health and standard of living. It uses micro data from household surveys, and—unlike the Inequality-adjusted Human Development Index—all the indicators needed to construct the measure must come from the same survey. More details can be found in Alkire and Santos (2010).

Methodology

Each person is assigned a deprivation score according to his or her household's deprivations in each of the 10 component indicators. The maximum score is 100%, with each dimension equally weighted; thus the maximum score in each dimension is 33.3%. The education and health dimensions have two indicators each, so each component is worth 33/2, or 16.7%. The standard of living dimension has six indicators, so each component is worth 33.6/6, or 5.6%.

The thresholds are as follows:

- Education: having no household member who has completed five years of schooling and having at least one school-age child (up to grade 8) who is not attending school.
- Health: having at least one household member who is malnourished and having had one or more children die.
- Standard of living: not having electricity, not having access to clean drinking water, not having access to adequate sanitation, using “dirty” cooking fuel (dung, wood or charcoal), having a home with a dirt floor, and owning no car, truck or similar motorized vehicle while owning at most one of these assets: bicycle, motorcycle, radio, refrigerator, telephone or television.

To identify the multidimensionally poor, the deprivation scores for each household are summed to obtain the household deprivation, c . A cut-off of 33.3%, which is the equivalent of one-third of the weighted indicators, is used to distinguish between the poor and nonpoor. If c is 33.3% or greater, that household (and everyone in it) is multidimensionally poor. Households with a deprivation score greater than or equal to 20% but less than 33.3% are vulnerable to or at risk of becoming multidimensionally poor. Households with a deprivation score of 50% or higher are severely multidimensionally poor.

The MPI value is the mean of deprivation scores c (above 33.3%) for the population and can be expressed as a product of two measures: the multidimensional headcount ratio and the intensity (or breadth) of poverty.

The headcount ratio, H , is the proportion of the population who are multidimensionally poor:

$$H = \frac{q}{n}$$

where q is the number of people who are multidimensionally poor and n is the total population.

The intensity of poverty, A , reflects the proportion of the weighted component indicators in which, on average, poor people are deprived. For poor households only (c greater than or equal to 33.3%), the deprivation scores are summed and divided by the total number of poor persons:

$$A = \frac{\sum_1^q c}{q}$$

where c is the deprivation score that the poor experience.

The deprivation score c of a poor person can be expressed as the sum of deprivations in each dimension j ($j = 1, 2, 3$), $c = c_1 + c_2 + c_3$.

The contribution of dimension j to multidimensional poverty can be expressed as

$$Contrib_j = \frac{(\sum_1^q c_j)/n}{MPI}$$

Example using hypothetical data

Indicator	Household				Weights
	1	2	3	4	
Household size	4	7	5	4	
Education					
No one has completed five years of schooling	0	1	0	1	$\frac{1}{3} \div 2$ or 16.7%
At least one school-age child not enrolled in school	0	1	0	0	$\frac{1}{3} \div 2$ or 16.7%
Health					
At least one member is malnourished	0	0	1	0	$\frac{1}{3} \div 2$ or 16.7%
One or more children have died	1	1	0	1	$\frac{1}{3} \div 2$ or 16.7%
Living conditions					
No electricity	0	1	1	1	$\frac{1}{3} \div 6$ or 5.6%
No access to clean drinking water	0	0	1	0	$\frac{1}{3} \div 6$ or 5.6%
No access to adequate sanitation	0	1	1	0	$\frac{1}{3} \div 6$ or 5.6%
House has dirt floor	0	0	0	0	$\frac{1}{3} \div 6$ or 5.6%
Household uses “dirty” cooking fuel (dung, firewood or charcoal)	1	1	1	1	$\frac{1}{3} \div 6$ or 5.6%
Household has no car and owns at most one of: bicycle, motorcycle, radio, refrigerator, telephone or television	0	1	0	1	$\frac{1}{3} \div 6$ or 5.6%
Results					
Household deprivation score, c (sum of each deprivation multiplied by its weight)	22.2%	72.2%	38.9%	50.0%	
Is the household poor ($c > 33.3\%$)?	No	Yes	Yes	Yes	

Note: 1 indicates deprivation in the indicator; 0 indicates nondeprivation.

Weighted deprivations in household 1:

$$(1 \cdot 16.67) + (1 \cdot 5.56) = 22.2\%.$$

Headcount ratio (H) =

$$\left(\frac{7 + 5 + 4}{4 + 7 + 5 + 4} \right) = 0.800$$

(80% of people live in poor households).

Intensity of poverty (A) =

$$\frac{(72.2 \cdot 7) + (38.9 \cdot 5) + (50.0 \cdot 4)}{(7 + 5 + 4)} = 56.3\%$$

(the average poor person is deprived in 56.3% of the weighted indicators).

$$MPI = H \cdot A = 0.8 \cdot 0.563 = 0.450.$$

Contribution of deprivation in:

Education:

$$Contrib_1 = \frac{16.67 \cdot 7 \cdot 2 + 16.67 \cdot 4}{4 + 7 + 5 + 4} / 45.0 = 33.3\%$$

Health:

$$Contrib_2 = \frac{16.67 \cdot 7 \cdot 5 + 16.67 \cdot 4}{4 + 7 + 5 + 4} / 45.0 = 29.6\%$$

Living conditions:

$$Contrib_3 = \frac{5.56 \cdot 7 \cdot 4 + 5.56 \cdot 4 \cdot 3}{4 + 7 + 5 + 4} / 45.0 = 37.1\%$$

Calculating the contribution of each dimension to multi-dimensional poverty provides information that can be useful for revealing a country's configuration of deprivations and can help with policy targeting.

Notes

- 1 The inequality aversion parameter affects the degree to which lower achievements are emphasized and higher achievements are de-emphasized.
- 2 A_x is estimated from survey data using the survey weights,

$$\hat{A}_x = 1 - \frac{X_1^{w_1} \dots X_n^{w_n}}{\sum_1^n w_i X_i}, \text{ where } \sum_1^n w_i = 1.$$

However, for simplicity and without loss of generality, equation 1 is referred to as the Atkinson measure.